Changing the Equation:
After-School Math Curriculum

ART MATH

Based on *After-School Math PLUS*
from the Educational Equity Center at
Academy for Educational Development

¡explora!
Ideas You Can Touch
Ideas que puedes tocar

Funding provided by
Sandia National Laboratories

¡explora!
Ideas You Can Touch
Ideas que puedes tocar

KIT: ART MATH

This After-School Math Kit includes engaging activities that are fun for both students and after-school staff. Students learn math while working in cooperative, supportive groups facilitated by staff members. Even better, after-school staff members don't need to be math experts!

Through the activities in this Kit, students and staff hone math skills, gain confidence in math, and increase their enjoyment of math.

THIS KIT INCLUDES

■ **Instructions to facilitate 3 activities.** These activities are simplified summaries and extensions of the "After-School Math PLUS: Art Math" curriculum created by the Educational Equity Center at Academy for Educational Development (AED).
The 3 activities are:
Part 1. Creating a Kaleidoscope
Part 2. Fun with Geometry
Part 3. Fractal Art

■ **Scientist Spotlight** Ask your students "who is a scientist?" and you'll typically get answers that include white lab coats, microscopes, and bubbling beakers. All of these images reflect some aspect of science and STEM (Science, Technology, Engineering, Math), but they don't provide a full picture. We include stories of two STEM professionals that work at Sandia National Laboratories. Read these with your students, and together list some of the activities, skills, and experiences from the stories. What surprises your students? What was unexpected? Does this change how they think of scientists and engineers?

TIPS FOR LEADING ACTIVITIES

- Give students opportunities to share their ideas with you and with each other.
- Make sure to introduce each activity. Learning happens best when learners know what's coming up and why it matters to them.
- Ask open-ended questions, rather than those that have a "yes" or "no" answer.
- Ask questions that inspire the learner to thoughtfully analyze a situation and consider consequences, such as, "What do you think will happen if you do this?"
- Give the learner time to answer the question. Ask the question, then wait. A while. Trust us: thoughtful answers take time.
- When a learner tells you what they think, respond by repeating and paraphrasing what they have said without criticism.
- Don't give too much praise or reject ideas. Telling a learner they are right or wrong can discourage them from generating additional ideas or pursuing deeper exploration.

Part 1 - Creating A Kaleidoscope **Main Idea:** Students will use rulers, compasses, and protractors to place three mirrored surfaces at specific angles to create complex, symmetrical reflections of colorful patterns in a kaleidoscope. They will then explore the math involved in the construction and in the reflected patterns.

INTRODUCTION

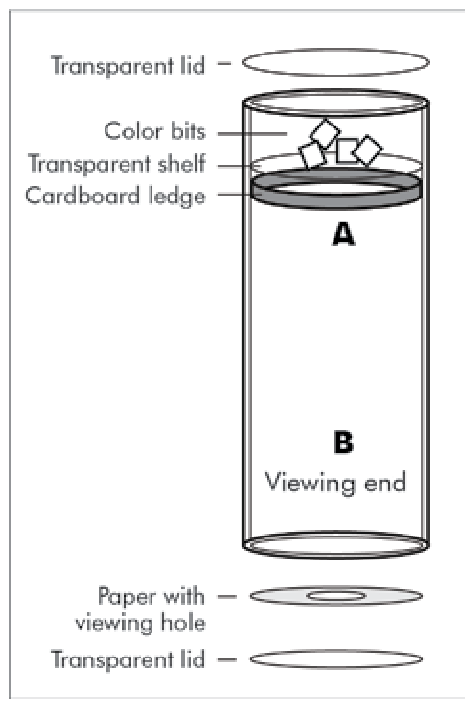
When light enters a kaleidoscope, the mirrored surfaces bounce the light back and forth (think of a ball bouncing off a wall), creating multiple images of the beads, sequins, and other objects inside. Students will discover the relationship between the number of images they see and the position of the mirrors. For instance, a two-mirror kaleidoscope with mirrors positioned in a V-shape with a 45-degree angle will create eight reflections, or a four-pointed star. However, if the two mirrors are situated at 90 degrees, then four images, or a two-pointed star, will appear. A 30-degree position will create twelve images, or a six-pointed star.

MATERIALS

For each student:
- 1 empty can with both ends removed - this tube will be the body of the kaleidoscope
- Construction paper
- 1 sheet cardboard mirror
- 1 sheet transparency film
- Clear colored beads or plastic chips
- 1 piece of corrugated cardboard

For each small group:
- Markers
- Ruler
- Compass
- Protractor
- Clear tape
- Scissors
- Glue sticks
- 4 Plexiglass mirrors

Tube construction



ACTIVITY: CREATING A KALEIDOSCOPE

Note: In preparation, make a kaleidoscope before the class for demonstration and comparison. Make this kaleidoscope with two mirrors rather than three. Initially, put three mirrors together in a triangle as per the instructions below, but blacken one of the mirrors or simply turn it around, so that only two of the mirrors can reflect the image.

¡explora!
Ideas You Can Touch
Ideas que puedes tocar

STEP 1: PREPARE THE PIECES (5 MINUTES)

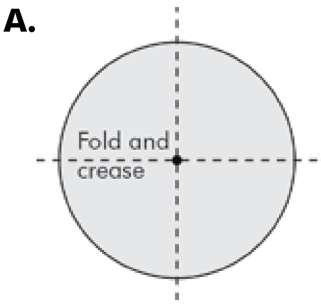
Preparing the tube: Have students mark one end of their can side A and the other end B.

Paper circle: Set one end of the can on a piece of construction paper and trace the end with a pencil to make a circle. Cut out the circle. This is now a template.

Large transparent circles: Use this template to cut two circles out of the transparency film. To ensure the paper circle template doesn't move, use tape to secure the center of the paper circle to the transparency film, then cut out two circles this size.

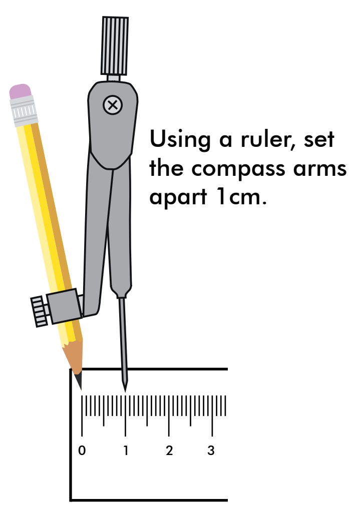
A. *Paper with viewing hole:* Fold the paper circle template in half. The crease at the center of the circle is the diameter. Use a ruler to measure the crease and put a mark at the center. Now unfold it.
B. Set the compass at 1cm.
C. Hold the compass point on the center mark, then carefully draw a 1cm-radius circle.
D. Fold the paper circle in half again and cut along your line to make a hole in the center.

A.

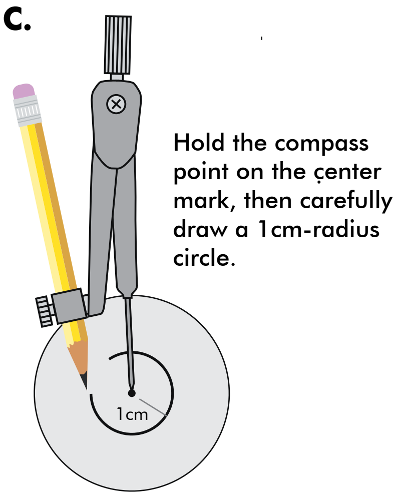


Small transparent circle: Place the ends of the compass just inside the end of the can tube and adjust it so that the pencil and metal point just barely touch the inside edges of the tube. Then, without moving the ends, transfer the compass to a piece of paper. Press firmly with the metal point and make a small mark with a pencil on the paper to leave two marks. Use a ruler to draw a line between the two marks. This is the diameter of a new circle. Find the radius by cutting the diameter in half. Put a mark on the diameter line in the middle. Place the point of the compass on the center mark, put the pencil at the end of the radius, and draw a circle. Cut out this circle, tape it on the transparency film, and cut out the circle in the film. This circle will fit inside the tube.

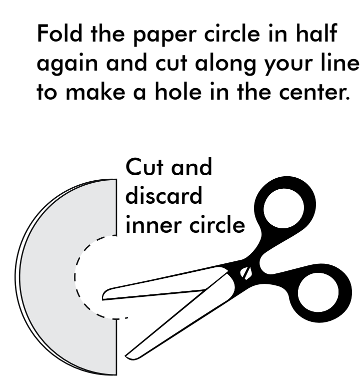
B.



C.



D.



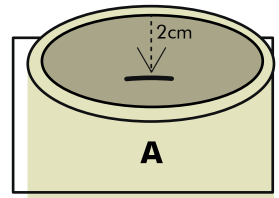
Funding provided by Sandia National Laboratories

¡explora!
Ideas You Can Touch
Ideas que puedes tocar

Cardboard strip: Cut a strip of corrugated cardboard 20-30cm long and 1cm wide. Make two of these. In some cases, you may need to make the strip longer depending on the circumference of your can.

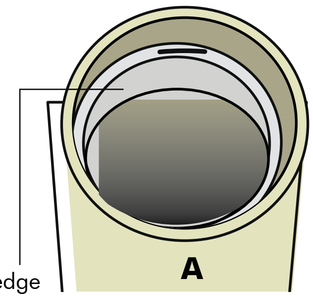
STEP 2: ASSEMBLY (10 MINUTES)

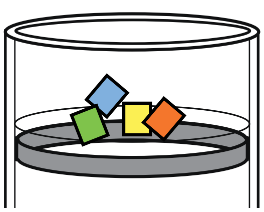
Tube assembly: Place a ruler 2cm into end A of the tube. Make a mark on the tube at the end of the ruler. Continue to make five more marks, all 2cm in from the end of the tube.



Roll the cardboard strip into a loose circle and insert it into the end of the tube marked A. Push the strip up against the tube ensuring the cardboard ring is snug and touches the can all the way around. The ends will overlap; mark the strip where the edges overlap and cut off the extra.

Apply glue just under the pencil marks inside end A. Insert the cardboard strip so that it covers the glue. Adjust the strip so that the top edge just touches the pencil marks and press into place.



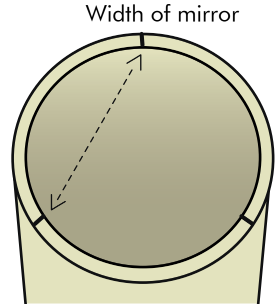


Take the small transparent circle, insert it into end A, and let it rest on the cardboard ring. Place colored transparent beads and plastic chips on the transparent circle.

Apply glue to the ridge of end A and press one of the large transparent circles firmly in place on the outside of the tube. Instead of glue you could tape this circle into place.

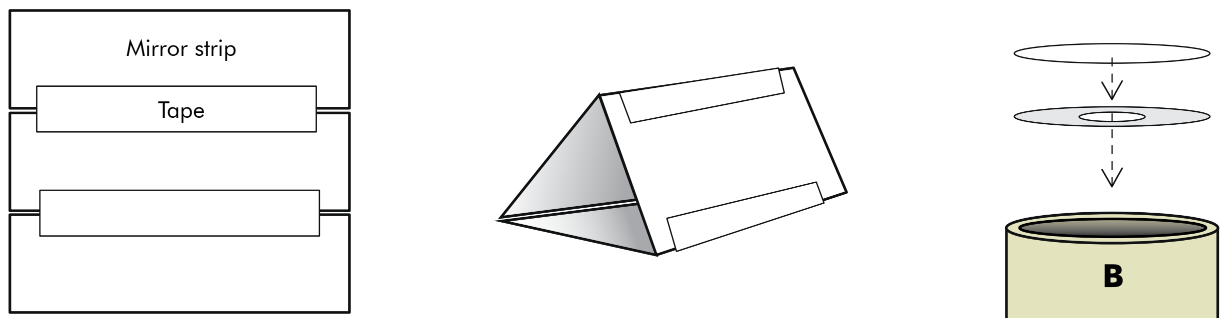
STEP 3: THE MIRRORS (5 MINUTES)

Cardboard mirror strips: Use the other cardboard strip and wrap it around the outside of the tube. Mark where the strip meets itself and cut off the excess. Measure the length of the strip and divide this length by three. Use this number to mark the strip to make three equal parts. Wrap the strip around tube end B, hold it in place, and make marks on the end of the tube that correspond to the marks on the strip. Make a mark where the ends of the cardboard strip meet. Using these marks, measure the distance from one mark to another; the distances between any two marks should be equal. This distance will be the width of the mirror strips.



Insert a ruler inside end B until it touches the cardboard ring at the other end. Read this distance on the ruler. This distance will be the length of the mirror strips.

Using the measured width and length, measure and cut out three mirror strips. Place the strips mirrored-side down and join them together lengthwise and tape them together. Stand the strips up and fold them in so that the mirrored sides form the inside of a triangle. Insert the triangle into the tube from end B until it touches the ring at the other end.



Apply glue to the ridge at end B, attach the paper circle with the hole in the center, and cover it with the last large transparent circle.

Your kaleidoscope is ready to use!

ACTIVITY: EXPLORING WITH THE KALEIDOSCOPE AND MIRRORS

STEP 1: EXPLORE THE KALEIDOSCOPE (20 MINUTES)

Exploring with the kaleidoscope: Have students look through their kaleidoscopes. What do they see? What works well? Share them with one another. Pass around the two-mirrored kaleidoscope for comparison. How many reflections does each yield?

Discussion: Bring students together without the kaleidoscopes. Have students try to explain how the kaleidoscope works: What is going on? How does your kaleidoscope differ from the two-mirrored kaleidoscope? Record responses on a white board or chart paper so everyone can see.

STEP 2: EXPLORING MIRRORS (15 MINUTES)

Materials: 4 Plexiglass mirrors, each 5 inches x 7 inches

1. Stand two mirrors about 12 inches apart. Place an object between them. How many reflections do you see?

Funding provided by Sandia National Laboratories

2. Tape two mirrors together – the tape should act like a hinge so that you can swivel the mirrors. Tape a protractor to the table so that the straight edge is perpendicular to the edge of the table. Place the mirrors so that the left mirror touches the straight edge of the protractor and adjust until the hinge lines up with the 0 mark on the protractor. Place a small object on the table at the open end of the mirrors.

   Holding the left mirror in place, move the right mirror. How many images can you make? What is the fewest number of images you can make?

   Find the angle of the mirrors for various mirror positions. Use the angle numbers that are on the inside of the protractor arc. At what angle do you get the largest number of images?

   For older students:

3. Calculate the number of images seen at any angle using the following formula: 360 divided by ___ angle, then subtract 1. Example: For an angle of 40°, 360 ÷ 40 = 9 – 1 = 8 images.

4. Predict the mirror angle that will show a certain number of images. Add 1 to the number of images and divide 360 by this number.
   Example: For 6 images, 6 + 1 = 7; 360 ÷ 7 = 51°

   Do an experiment to check. Set the right hand mirror at 51 degrees. How many images do you see? Is the mathematical prediction correct?

5. Practice predicting the number of images seen at different mirror angles or the mirror angle needed to see a certain number of images.

Part 2 - Fun with Geometry **Main Idea:** Geometry explores the math of shapes, such as squares, triangles, pentagons, and others. If shapes are similar in form and size, geometry can be used to create repeating patterns known as tessellations. By understanding the concept of symmetry, students can arrange tessellations and create unique, interesting art.

INTRODUCTION

A tessellation is created when one or more geometric shapes are repeated over and over again to fill a surface without any gaps or overlaps. Tessellations are used to tile floors and ceilings, to create mosaics, and more.

Preparation: Either use our tessellation examples or bring in some examples of tessellations. These could include photographs of different floor tile patterns, ceiling tiles, a checker board, M.C. Escher's "Butterflies," etc. Cut out squares and triangles in two colors of paper so that each student can cover a sheet of paper with either triangles or squares. Practice making a tessellation by alternating colors and positions of triangles on a sheet of paper.

MATERIALS

- Paper
- Pencils
- Art supplies
- Rulers
- Protractors
- Scissors
- Glue
- Tessellation examples (pages AM 11 and AM 12)

ACTIVITY: FUN WITH GEOMETRY

STEP 1: TESSELLATION EXAMPLES (10 MINUTES)

Show several examples of tessellations. Use the samples provided or ones you find in books at the library or online. Ask: "What do you notice about these works of art? What is special about this symmetrical pattern? (It can go on forever.) Where have you seen other types of symmetrical patterns?" (e.g., tiles or bricks on buildings and floors, public art). Tell students that these symmetrical patterns are called tessellations.

Hold up a triangle and ask: "What do you think it means that these triangles are symmetrical?" Try to get each student to express an idea. (Symmetry: one side exactly mirrors the other). Then demonstrate how to make a tessellation by making a triangle tessellation. Reiterate that the triangles are symmetrical and tessellations are symmetrical patterns.

STEP 2: TESSELLATIONS (20 MINUTES)

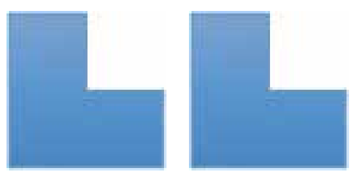
Give each student a sheet of paper, a glue stick, and a handful of the squares or triangles in both colors.

Starting at the center of the paper, glue down a triangle or a square. Next, add another shape, but in the other color. Continue this process until the entire page is covered. Pieces may stick out beyond the edge of the paper.

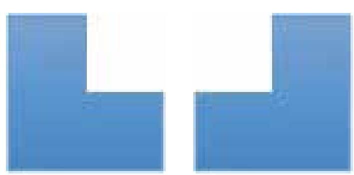
Discussion: If you had a larger paper, could you continue this pattern? Could you cover the side of a building? What is special about this pattern? (It goes on forever.) This continuous repetition is a key feature of a tessellation. Could you arrange the shapes in a different way and still have a tessellation? Point out the photographs or other examples. How are they similar?

ACTIVITY: SYMMETRY TRANSFORMATION TYPES

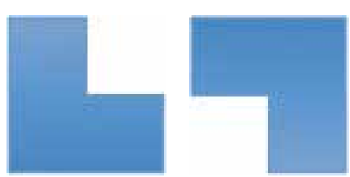
There are many ways that a single pattern can be repeated. If you look closely at many tessellations, you'll see figures using these transformation techniques: translation, reflection, and rotation, or a combination of any of these.



Translation (above) – This is a type of symmetry in which the shape is simply copied and repeated. This technique is called "slide" or "glide" or "translate." One shape is translated over and over.



Reflection (above) – The shape is repeated, but is facing a different way.



Rotation (above) – The shape is repeated, but is turned around a point.

Combination: You can create a tessellation that combines all three of these patterns. You can create one row doing translation, then the next row doing reflection, or have a rotational tile that repeats along a row.

¡explora!
Ideas You Can Touch
Ideas que puedes tocar

ACTIVITY: CREATING TESSELLATION ART

STEP 1: MAKE ART (20 MINUTES)

Have students pick a simple shape or figure from at least one transformation type above. Place the shape or figure to tessellate on a sheet of paper and trace it. Slide the shape over, line it up with the first shape so there are no overlaps or gaps, and trace it again. Continue tracing the shape until the entire paper is covered. Use crayons or markers to color the shapes or figures. Can you color the page in a way that makes a pattern?

STEP 2: SHARE THE ART (10 MINUTES)

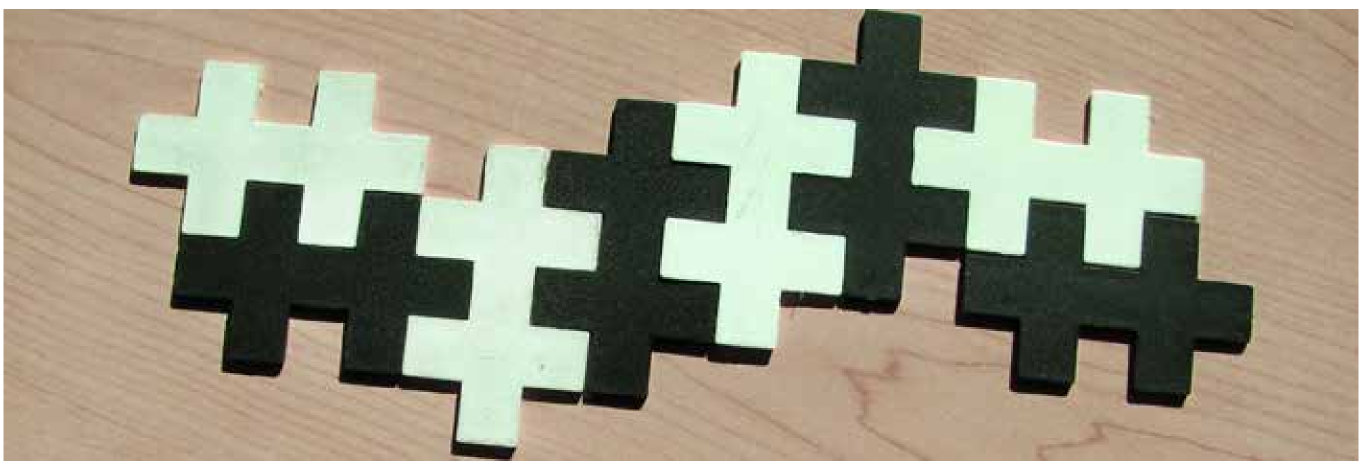
Post the student art on the wall and gather students for a discussion.

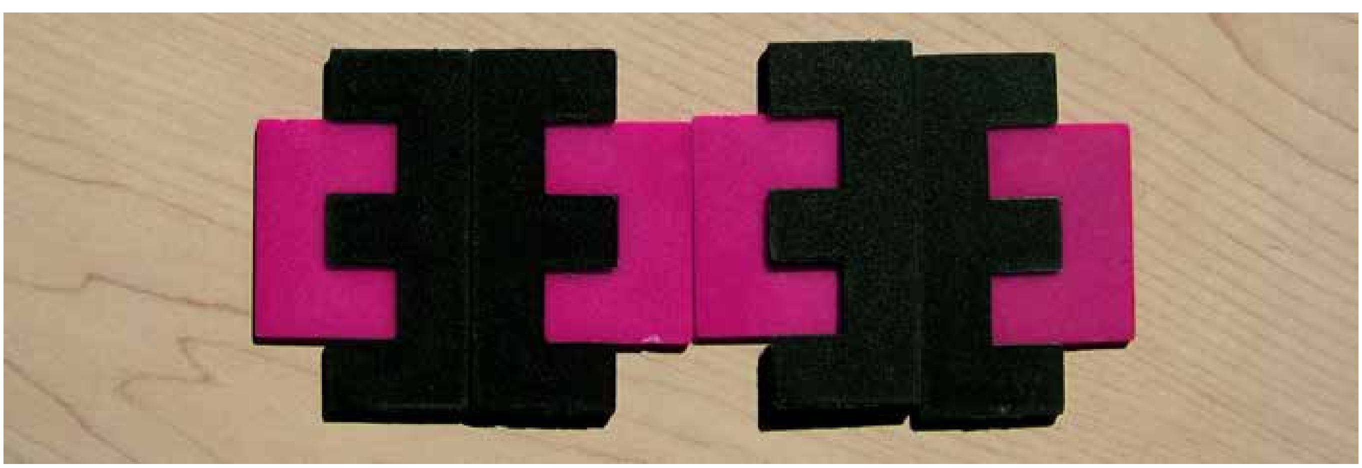
Discussion: Why did you do what you did? How do the pictures you made compare to what you see in a kaleidoscope?

Funding provided by Sandia National Laboratories

Ideas You Can Touch
Ideas que puedes tocar

Tessellation Examples



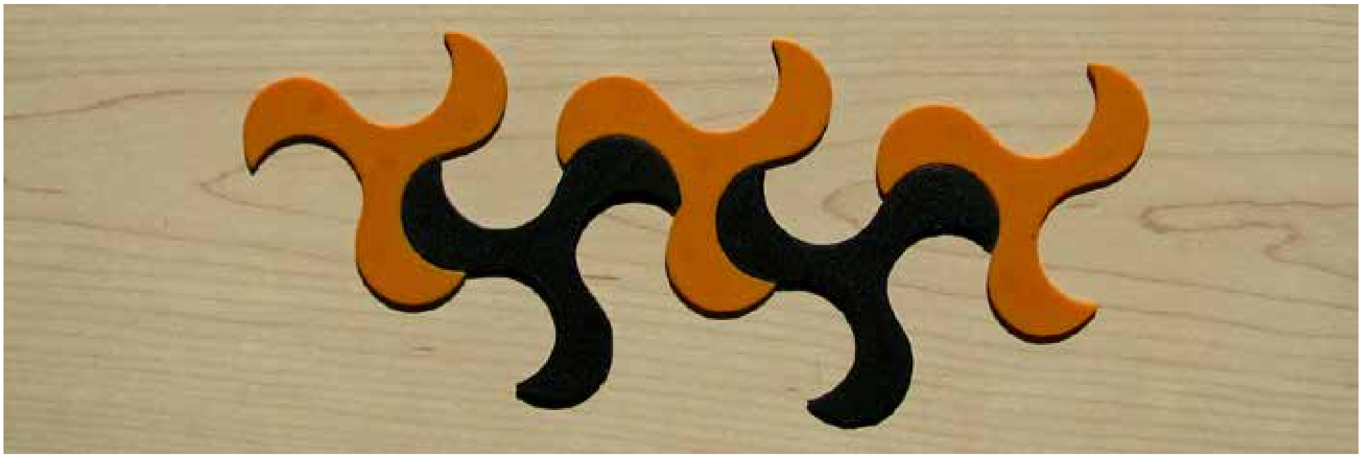


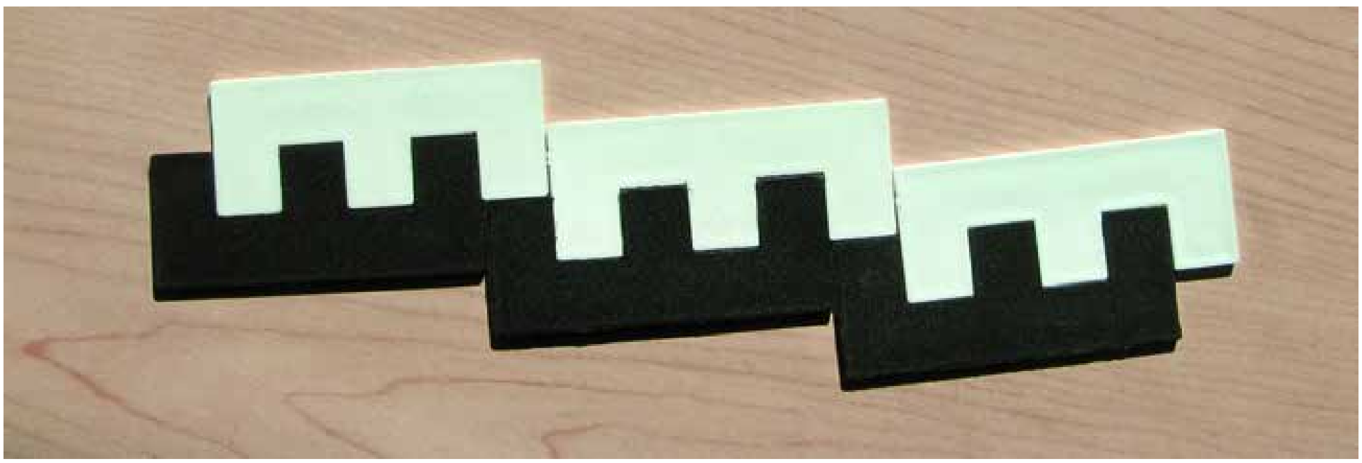
Based on *After-School Math PLUS* from the Educational Equity Center at Academy for Educational Development
Funding provided by Sandia National Laboratories

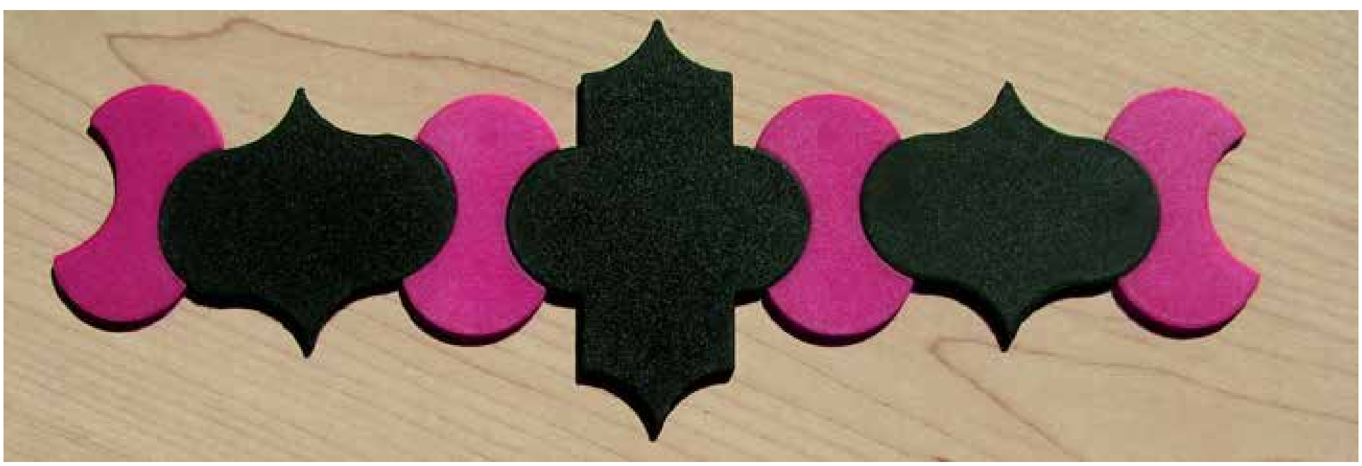
¡explora!

Ideas You Can Touch
Ideas que puedes tocar

Tessellation Examples







Funding provided by Sandia National Laboratories

Part 3 - Fractal Art Main Idea: ***Fractals*** are never-ending, self-similar patterns; they repeat themselves on different scales. ***Geometric fractals*** are made by following a simple process over and over again to generate the pattern. Using measurement and precision, students will generate their own *Sierpinski triangles* that are interesting art pieces alone, but can also be combined to make larger fractal art in groups of 3, 9, and 27.

INTRODUCTION

Fractals are commonly found in nature, geometry, and algebra. Two common fractal patterns found in nature are branches and spirals. Today, students will generate a geometric fractal called a ***Sierpinski triangle*** **(examples are easily found online)**. It is made of ***equilateral triangles*** (triangles in which all three sides are equal). The midpoints are connected to make a new, smaller triangle inside of the larger one and that new triangle is then removed from that generation, and the process continues in the remaining triangles. The removed triangle will always be pointing down while the rest will be pointing up. The number of triangles pointing up will increase by a factor of three in each new generation.

Preparation: Make copies of the triangle handout on page AM 15. Practice making a Sierpinski triangle with four generations. If possible, find and share fractal examples.

MATERIALS

- Triangle template (see page AM 15)
- Sierpinski carpet and Koch curve samples (see page AM 16)
- #2 pencils and erasers
- Crayons, colored pencils, or markers
- Rulers (1 per student)
- Scissors

ACTIVITY: EXPLORING FRACTALS

STEP 1: WHAT DO FRACTALS LOOK LIKE? (10 MINUTES)

Discussion: Gather images of fractals from nature and math. Excellent math examples include the Mandelbrot set and Koch curve. These can be found in books at the library or online. Show several examples of fractals from nature. Ask students to describe what they see. How are these fractals? What shape is repeating? Show examples of fractals from mathematics. How are these fractals? What shape is repeating?

Fractals are ***self-similar***. Does anyone know what that means? It means the parts are the same as the whole. As you zoom in on a fractal and look at a smaller piece of it, it will have the same shape and pattern as the whole and as every other part. (Demonstrate using Koch curve or Sierpinski carpet).

ACTIVITY: SIERPINSKI TRIANGLES

STEP 1: FIND THE MIDPOINTS (5 MINUTES)

Give each student a pencil, a ruler, and a triangle template. Using the ruler, students should measure each side of their triangle and put a dot or a small line at the midpoint of each side. Once all of the midpoints are marked, students then use a ruler to draw a line connecting the midpoints to form a new triangle in the middle of the big one.

STEP 2: COLOR IN THE TRIANGLE (5 MINUTES)

Using colored pencils, crayons, or markers, have students color in the upside-down triangle they just made. This is the triangle that is being removed and is no longer included in the number of triangles or area calculations.

Now, there are three triangles remaining. Find the midpoints and draw in the downward-facing triangles for each.

STEP 3: REPEAT FOR FOUR GENERATIONS (10 MINUTES)

Color in the three new downward-facing triangles. Encourage students to be creative and use different materials, colors, and techniques to fill in each generation of triangles. Repeat the process two more times until students have completed four generations of their fractals. After the last generation, they can color in the remaining upward-facing triangles too, so their whole triangle is colored in.

Discussion: How many triangles are in the second generation of the Sierpinski triangle? Is it possible to calculate how many triangles are in each stage rather than counting?

STEP 4: COMBINE TO MAKE A BIGGER FRACTAL (10 MINUTES)

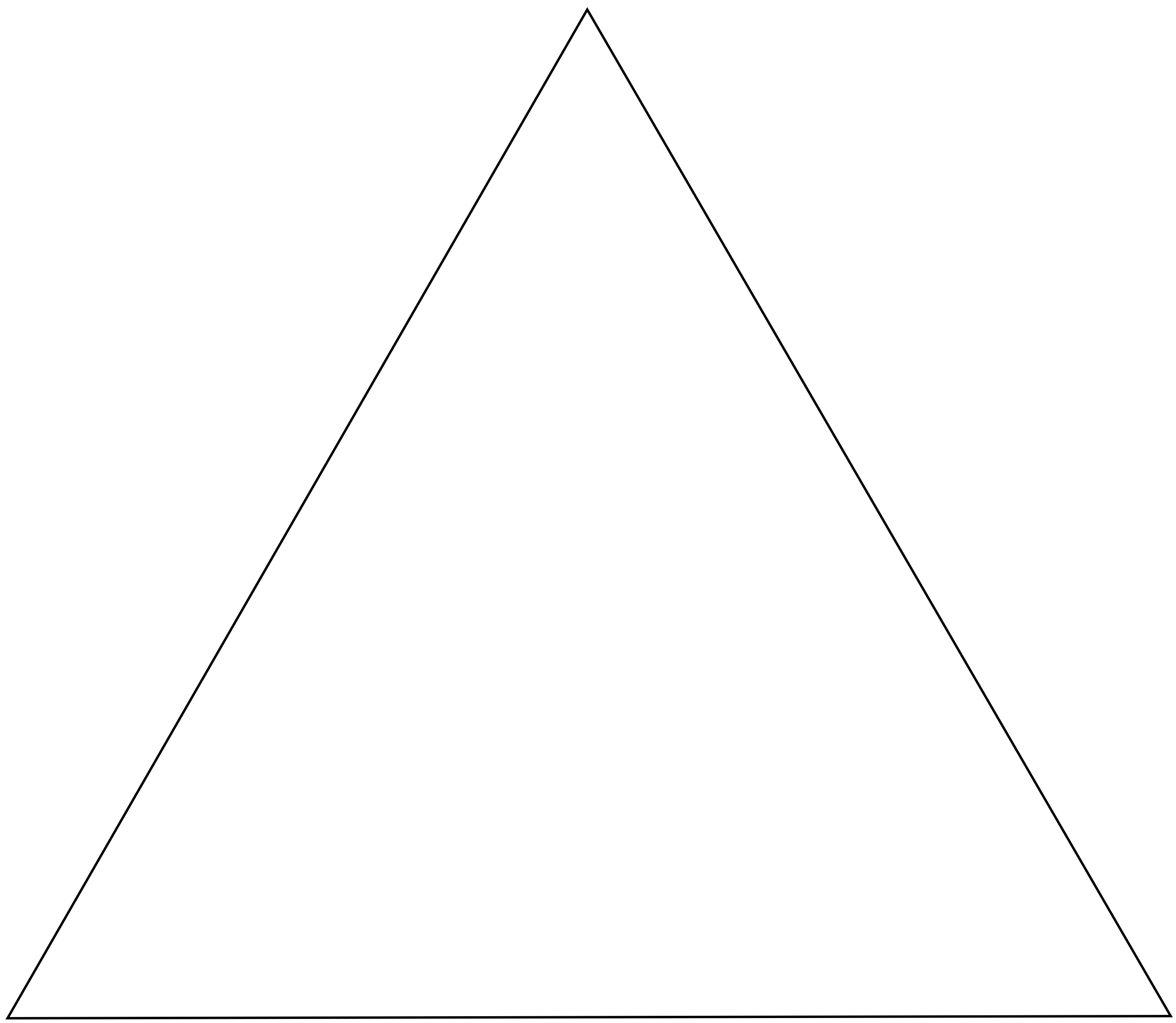
Have each student cut out his or her fractal. Ask students if they can make bigger triangle fractals by combining with their peers. How many students need to join their triangles to make another Sierpinski triangle? What's the biggest triangle the class can make?

Have students break into groups of 3 and join their triangles to make new, bigger fractals. If there are enough students, join 3 of the group's triangles to form an even bigger triangle of 9 student triangles. How many student triangles would it take to make an even bigger version than the 9-triangle fractal?

Hang the group fractals (of 3, 9, or 27 student triangles) on the wall and have students look at them from a distance.

Discussion: What does it look like? Is it art? Is it math? Ask students to verbalize how math can be used to make interesting art.

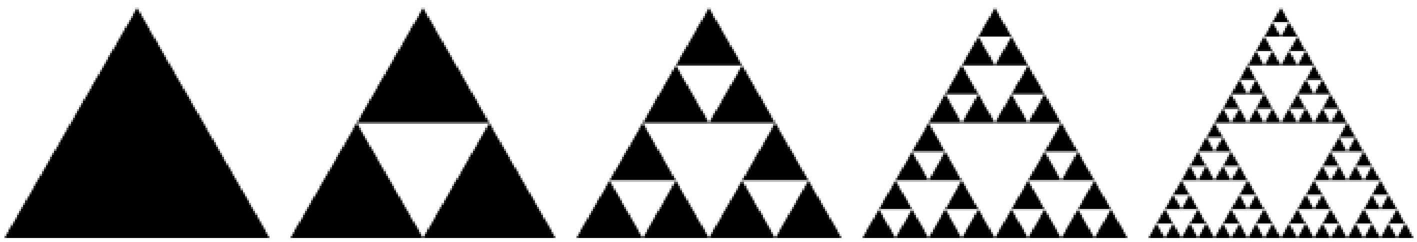
Triangle Template



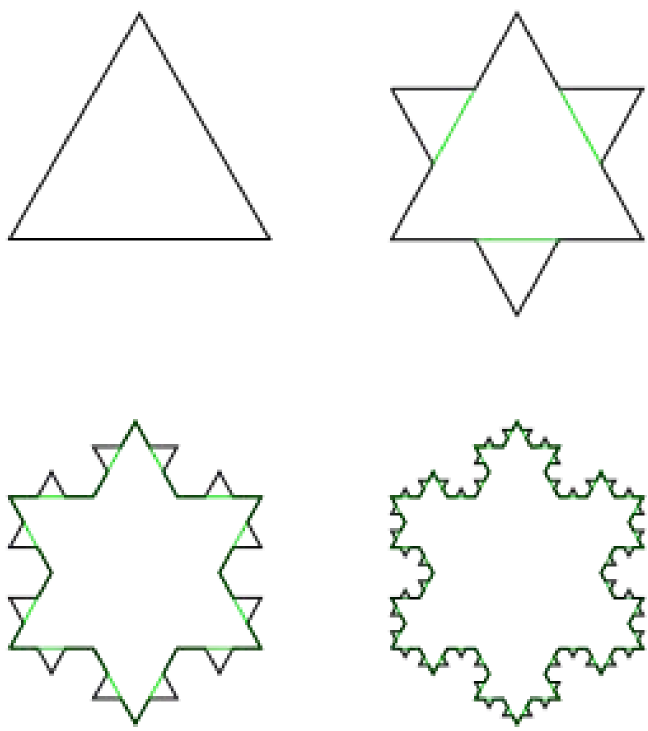
Based on *After-School Math PLUS* from the Educational Equity Center at Academy for Educational Development
Funding provided by Sandia National Laboratories

¡explora!
Ideas You Can Touch
Ideas que puedes tocar

Sierpinski carpet and Koch curve samples



Sierpenski Carpet, which is made of many Sirepenski Triangles



Koch Snowflake which includes many Koch curves forming all the edges

Funding provided by Sandia National Laboratories

I am a creative designer at Sandia National Laboratories, which means I help scientists communicate their work by designing posters, brochures, reports, and presentations. I like my job because I get to work with people from all over Sandia, learn about their work, and then help them share it with other people. In school, I knew I wanted to use my creativity to help people, but I knew I'd need math skills, too. All the math classes I took—even though they were hard—helped me learn to think logically and taught me many skills that inform my design work. I use geometry and proportions every day to come up with interesting new designs in different sizes that attract people's attention. Drawing pictures really helped me in my math classes, because I'm a visual learner, so picturing the math helped me remember it. I didn't always like math in school, but I'm glad I stuck with it, because I use it every day!

STACEY LONG
Creative Designer

I am a civil engineer at Sandia National Laboratories, working in renewable energy. I help American Indian tribes and Alaskan Native villages learn about and take advantage of renewable energy technologies, like solar panels. When I was in elementary school, I liked math and science and always wanted to figure out how different gadgets worked. Now, I get to educate others about our natural resources and how renewable energy technologies work. Math isn't always easy, but it is very useful and is part of what helped me become an engineer. Engineering is a great way to help people! I had to practice my math and engineering skills a lot to get to the point that I can help to educate others. Keep practicing your math skills, and you can find an exciting and fulfilling career!

SANDRA BEGAY
Civil Engineer

Funding provided by Sandia National Laboratories